Indiana State Math Contest 2019

Algebra I

This test was prepared by faculty of
Indiana University, Bloomington
Jenny Cox, Associate Instructor

Do not open this test booklet until you have been
advised to do so by the test proctor.

1. Solve the quadratic equation $2x^2 = 5x + 3$.
(a) 1 and 3/2   (b) -1 and -3/2   (c) -5 and -1/2   (d) 1/2 and -3   (e) -1/2 and 3

2. Solve the equation for m: $0.2(8 - m) = \frac{1}{5}(10m + 3) - 0.4$
(a) 11/7   (b) 7/11   (c) 13/11   (d) 11/13   (e) infinitely many solutions

3. Find the product: $(x - 3)^2(x + 3)^2$
(a) 1   (b) $x^4 + 18x^2 + 81$ (c) $x^4 - 81$ (d) $x^4 - 18x^2 + 81$ (e) $x^4 + 81$

4. If $f(x) = x^3 - 2x + 1$, find f(-2).
(a) 13   (b) -11   (c) -3   (d) 11   (e) 3

5. Find the maximum y-value for the graph $y = -2x^2 - 8x + 15$.
(a) 23   (b) -2   (c) -9   (d) 15   (e) no maximum

6. Solve the equation: $\sqrt{4x + 3} + 6 = 10$.
(a) 1/4   (b) 13/4   (c) 8   (d) -1/8   (e) 23/8

7. Solve the inequality $\frac{1}{2} - x < 3x + \frac{3}{4} \le x + \frac{5}{6}$ for x.
(a) $-16 < x \le \frac{1}{24}$ (b) $-\frac{1}{16} < x \le \frac{1}{24}$ (c) $\frac{1}{24} < x \le \frac{1}{16}$ (d) $-\frac{1}{24} < x \le \frac{1}{16}$ (e) no solution

8. Simplify the root: $\sqrt[3]{81x^6y^8}$
(a) $3\sqrt[3]{x^6y^8}$ (b) $3x^2\sqrt[3]{3y^8}$ (c) $x^2\sqrt[3]{81y^8}$ (d) $27x^2y^2\sqrt[3]{3y^2}$ (e) $3x^2y^2\sqrt[3]{3y^2}$

9. Solve for x: $2x(x - 7) = 3(2 - 5x)$.
(a) 2 and -3/2   (b) $\frac{3\pm\sqrt{7}}{2}$ (c) $\frac{29\pm\sqrt{889}}{4}$ (d) -2 and 3/2   (e) 20/17

10. Simplify the expression: $\frac{(2x^3y)(4x^{-2}y^3)}{16x^5y^0}$
(a) 1   (b) $\frac{y^4}{2x^4}$ (c) $-\frac{y^4}{2x^4}$ (d) $\frac{y^4}{2x^5}$ (e) $-\frac{1}{x^4}$

11. The expression $x^{4a} - 9y^{2b}$ is equivalent to which of the following:

(a) $(x^{2a} + 3y^b)(x^{2a} - 3y^b)$ (b) $(x^{2a} + 3y^b)^2$ (c) $(x^{2a} - 3y^b)^2$

(d) $(x^{2a} + 9y^b)(x^{2a} - 9y^b)$ (e) $(x^{2a} - 9y^b)^2$

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12. Solve for a in the system of equations $\begin{cases} \frac{3}{5}a + 2b = b - 9 \\ 6a + \frac{7}{3}b = 3a + 3 \end{cases}$

(a) 25 (b) -18 (c) -45 (d) 15 (e) 18

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13. Find the equation of the line in general form with a slope perpendicular to line m and the same y-intercept as line n.

$m: \frac{4}{3}x - \frac{2}{5}y = \frac{1}{2}$

$n:\ 5y - 30 = 10x$

(a) 10/3x + y = -18 (b) x- 3y = 6 (c) 10x − 3y = -18 (d) 3x − 10y = -60 (e) 3x + 10y = 60

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14. Andy, Claude, Edgar, Frida, Georgia, Henri, and Jackson are choosing a seat from a row of 7 chairs. How many seating arrangements can be made if Andy and Frida insist that they must sit next to each other?

(a) 720 (b) 1440 (c) 4320 (d) 8640 (e) 5040

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15. Solve the absolute-value inequality $5|2 - 3x| + 8 \le -12$.

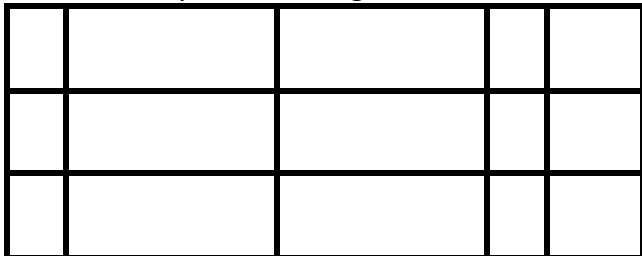(a) -3/2 ≤ x ≤ 2 (b) x ≥ 2 (c) x ≤ -3/2 or x ≥ 2 (d) x ≤ -3/2 (e) no solution

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16. Solve the equation $24x^2 - 5x - 36 = 0$ and find the sum of the two solutions.

(a) 5 (b)-5 (c) 59/24 (d) 5/24 (e) -5/48

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17. Simplify the expression: $\dfrac{(25^{\frac{1}{2}})(8^{-1/3})}{(16^{-\frac{1}{4}})(27^{\frac{1}{3}})}$

(a) 5/3 (b) 3/5 (c) 200/81 (d) 81/200 (e) 25/18

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18. How many total rectangles can be found in the following picture?

(a) 15 (b) 90 (c) 200 (d) 5040 (e) infinitely many

19. Simplify: $13(3^{-2} + 4^{-1})^{-1}$

(a) 1/36   (b) 169/36   (c) 36   (d) 13/36   (e) 130

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20. Determine the domain for the function $g(x) = \frac{\sqrt{x-4}}{|x-7|}$ in interval notation.

(a) $(-\infty, 7) \cup (7, \infty)$ (b) $(-\infty, 4) \cup (4, 7)$ (c) $[4, 7) \cup (7, \infty)$

(d) $(7, \infty)$ (e) $[4, \infty)$

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21. Find the sum of the solutions to $|4x + 1| = 3x + 5$.

(a) 4   (b) 22/7   (c) -6/7   (d) 34/7   (e) 1/3

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22. What is the x-intercept of the line containing the points (36, 17) and (-57, -34)?

(a) -85/31   (b) -10   (c) -5   (d) 5   (e) 10

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23. Two solutions of the equation Ax + By = 10 are (-2, 4) and (3, -5). Find A – B.

(a) 0   (b) 10   (c) 20   (d) = 24/5   (e) -3/5

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24. Find the distance between the two points (-1, 7) and (-3, 15).

(a) 4   (b) 68   (c) $2\sqrt{17}$ (d) 10   (e) $2\sqrt{15}$

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25. Find the negative solution for the equation $\sqrt{a^2 - 3a - 12} = 4$

(a) -4   (b) $\frac{3-\sqrt{57}}{2}$ (c) $\frac{3-\sqrt{65}}{2}$ (d) -7   (e) All solutions are positive

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26. How many unique diagonals can be drawn in a hexagon?

(a) 8   (b) 6   (c) 21   (d) 9   (e) 15

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27. A mixture of fruit punch and orange juice is created. The fruit punch contains 75% natural fruit juices and the orange juice contains 60% natural fruit juices. 10 pints of fruit punch is mixed with 20 pints of orange juice. What is the concentration of fruit juices in the resulting mixture?

(a) 65%   (b) 67.5%   (c) 62%   (d) 13.5%   (e) 6.5%

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28. If Jeff takes 6 hours to paint a bedroom and Ryan takes 8 hours to paint a bedroom, how long will it take the two of them when working together? Round to the nearest half hour.

(a) 3 hours   (b) 3.5 hours   (c) 4 hours   (d) 4.5 hours   (e) 7 hours

29. Find the median number of siblings for the students in a class, as shown in the bar graph below.
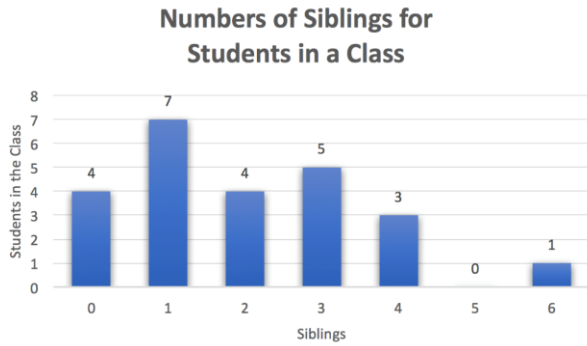
(a) 1   (b) 1.5   (c) 2   (d) 2.5   (e) 3

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30. A college student's grade point average is found by summing the total number of grade points (multiply credits by a numerical value for the grade) and then dividing by the total number of credits. An "A" is considered 4.0, a "B" is 3.0, a "C" is 2.0, a "D" is 1.0, and an "F" is 0. If a student received the following grades, what is his grade point average? Round to the nearest tenth.

Course	Credits	Grade
Calculus	4.0	A
Biology	5.0	B
Psychology	3.0	C
History	2.0	B

(a) 2.7   (b) 2.8   (c) 2.9   (d) 3.0   (e) 3.1

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31. Find the area of the shape below. Round to the nearest whole number.
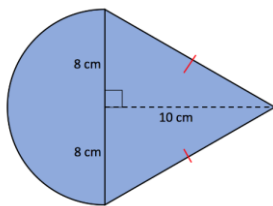
(a) 141   (b) 181   (c) 261   (d) 281   (e) 361

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32. A ball is thrown from a height of 20 feet. The height h of the ball in feet t seconds after it has been thrown is given by $h(t) = -16t^2 + 80t + 20$. After how many seconds will the ball hit the ground? Round to the nearest hundredth.

(a) 3.98 seconds   (b) 5.24 seconds   (c) 4.87 seconds   (d) 5.71 seconds   (e) 4.32 seconds

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33. Find the 101st term in the pattern 1, 4, 7, 10, …

(a) 3085   (b) 298   (c) 307   (d) 304   (e) 301

34. For the equation $4x^2 + 4x = 15$, which of the following is/are true:

I. The sum of the solutions is negative.

II. The product of the solutions is negative.

III. Exactly one of the solutions is a fraction.

(a) I only   (b) II only   (c) III only   (d) I and II only   (e) II and III only

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35. Solve the equation $7 - \sqrt{x} = \sqrt{5\sqrt{x} - 29}$.

(a) 169   (b) -36 and 36   (c) no solution   (d) 36 and 169   (e) 36

For the remainder of the test, assume all denominators are non-zero.

36. Solve the equation: $\dfrac{6x-1}{3x+5} = \dfrac{8x+3}{4x-1}$

(a) -14/39   (b) 14/39   (c) -14/59   (d) 14/59   (e) Infinitely many solutions

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37. Solve the formula for C: $\dfrac{3C}{A+C} + B = 4B$

(a) $C = \dfrac{AB}{1-B}$ (b) $C = \dfrac{AB}{1+B}$ (c) $C = \dfrac{A-B}{B}$ (d) $C = \dfrac{3AB}{1-3B}$ (e) $C = \dfrac{AB}{3-B}$

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38. Simplify the expression to its lowest terms: $\dfrac{6}{x} - \dfrac{2}{x+2} - \dfrac{3x+10}{x^2+2x}$

(a) $\dfrac{-3x-14}{x^2+2x}$ (b) $\dfrac{1}{x}$ (c) $\dfrac{x+2}{x^2+2x}$ (d) $\dfrac{x}{x+2}$ (e) $\dfrac{3}{x^2+2}$

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39. Multiply and divide $\dfrac{2x^2-9x-5}{4x^2+4x+1} \cdot \dfrac{6x^2+3x}{2x^2-7x-15} \div \dfrac{8x}{4x+6}$. Write the answer in simplest form.

(a) $\dfrac{12x^2}{4x^2+9}$ (b) $\dfrac{12x^2}{(2x+3)^2}$ (c) $\dfrac{3}{4x}$ (d) 3/4   (e) $\dfrac{6x}{8}$

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40. When dividing the polynomials $\dfrac{x^3-30x+19}{x-5}$, the remainder will be:

(a) -44   (b) 44   (c) -106   (d) 6   (e) -6