

# Indiana State Math Contest 

2017

## Algebra I

## This test was prepared by faculty at the University of Southern Indiana

Do not open this test booklet until you have been advised to do so by the test proctor.

1. Solve the equation for $x$ : $0.4(x-0.3)=1.6 x+7.8$
a. -6.6
b. -6.75
c. -7.5
d. -13.5
e. -20.25
2. Solve the equation for $x: \frac{x-3}{2}=\frac{x}{5}+6$
a. 3
b. 6
c. 7
d. 15
e. 25
3. Solve the inequality for $x$ : $\frac{1}{2}<\frac{3}{4}-x<\frac{5}{6}$
a. $(-1,1)$
b. $(-1,3)$
c. $(-5,2)$
d. $\left(-\frac{1}{12}, \frac{1}{4}\right)$
e. $\left(-\frac{10}{9}, \frac{2}{3}\right)$
4. Simplify the root $\sqrt[3]{24 x^{24}}$.
a. $8 x^{8}$
b. $2 x^{8} \cdot \sqrt[3]{3}$
c. $2 x^{2} \cdot \sqrt[3]{3 x^{3}}$
d. $2 x^{12} \cdot \sqrt[3]{6}$
e. $21 x^{21}$
5. Simplify the expression: $\frac{\left(2 x^{3} y^{2}\right)^{4}\left(3 x^{2} y\right)^{3}}{\left(6 x^{2} y^{3}\right)^{2}}$
a. $x^{4}$
b. $12 x^{14} y^{5}$
c. $2 x^{18} y^{4}$
d. $x^{15}$
e. $6 x^{3} y$
6. Factor $12 a^{4} b+15 a^{3} b^{3}-18 a^{2} b^{5}$
a. $9 a^{5} b^{-1}$
b. $3 a^{2} b\left(a+2 b^{2}\right)\left(4 a-3 b^{2}\right)$
c. $\left(6 a-2 b^{2}\right)\left(2 a+9 b^{2}\right)$
d. $\left(6 a+2 b^{2}\right)\left(2 a-9 b^{2}\right)$
e. $\left(4 a+3 b^{2}\right)\left(3 a-6 b^{2}\right)$
7. Simplify $(3 \sqrt{x}+\sqrt[3]{y})^{2}$
a. $9 x \cdot \sqrt[9]{y}$
b. $9 x+\sqrt[9]{y}$
c. $3 x+\sqrt[3]{y^{2}}$
d. $6 x+\sqrt[6]{y}$
e. $9 x+6 \cdot \sqrt[6]{x^{3} y^{2}}+\sqrt[3]{y^{2}}$
8. Solve the inequality $2 x-9>4$ or $9-2 x<4$.
a. $\left(-\infty, \frac{5}{2}\right)$
b. $\left(-\infty, \frac{13}{2}\right)$
C. $\left(\frac{5}{2}, \infty\right)$
d. $\left(\frac{13}{2}, \infty\right)$
e. $\left(-\infty, \frac{5}{2}\right) \cup\left(\frac{13}{2}, \infty\right)$
9. Solve for $x: 3 x(4-5 x)=6(x-7)$
a. $x=\frac{1 \pm \sqrt{71}}{5}$
b. $x=\frac{23}{8}$
c. $x=\frac{18}{7}$
d. $x=\frac{3 \pm \sqrt{114}}{15}$
e. $x=-1 \pm \sqrt{15}$
10. A disc golfer make a throw that traces the curve $f(x)=-.05 x^{2}+3 x+4$, where $f(x)$ is the height in meters and $x$ is the horizontal distance from the golfer in meters. About how far does the shot travel?
a. 4 m
b. 20 m
c. 30 m
d. 49 m
e. 61 m
11. Solve the equation $5 x y+4 y z=3 x z$ for $x$.
a. $x=-\frac{4 y z}{5 y-3 z}$
b. $x=-\frac{5 y}{2}$
c. $x=\frac{3(x y+y z)}{z}$
d. $x=5 y-4 y z-3 z$
e. $x=\frac{5 x y+4 y z}{3 z}$
12. At what value does $\frac{2 x}{3}+\frac{4 y}{5}=60$ cross the $x$-axis?
a. 0
b. 60
c. 90
d. 225
e. 450
13. What is the $y$-intercept of the line that passes through $\left(-\frac{4}{5},-\frac{3}{5}\right)$ and $\left(\frac{2}{5}, \frac{1}{5}\right)$ ?
a. $\left(0,-\frac{1}{15}\right)$
b. $\left(0,-\frac{1}{30}\right)$
c. $(0,0)$
d. $\left(0, \frac{1}{30}\right)$
e. $\left(0, \frac{1}{15}\right)$
14. Seven years ago, Angie's son was 3 years less than $1 / 3$ of her age. Now she is 10 years older than twice his age. How old was Angie when her son was born?
a. 40
b. 27
c. 25
d. 30
e. 42
15. The number of frogs in a pond is 60 less than one-third the number of turtles. If there are $T$ frogs in the pond, what expression represents the number of turtles?
a. $3 T+180$
b. $\frac{T}{3}-60$
c. $60-\frac{T}{3}$
d. $-60\left(\frac{T}{3}\right)$
e. $\frac{T}{3}+60$
16. An order of 3 burgers and 2 fries totals $\$ 24$. A second order of 7 burgers and 8 fries is $\$ 67$. How much does a burger and fries cost?
a. $\$ 8.00$
b. $\$ 9.10$
c. $\$ 10.20$
d. $\$ 11.30$
e. $\$ 12.40$
17. "The square of a number is greater than the number itself." is a true statement for which set(s) of numbers?
I. $x<0$
II. $0<x<1$
III. $x>1$
a. I and II only
b. I and III only
c. II and III only
d. All of the sets
e. None of the sets
18. Assume a linear relationship between the price of a concert ticket and the number sold. If a band sells 5000 tickets at $\$ 70$ each and 9500 at $\$ 45$ each, how many will they sell at a price of $\$ 60$ ?
a. 6800
b. 10800
c. 7800
d. 8800
e. 9800
19. Last week, the price of gas increased by $20 \%$. What percent does it need to decrease so that it is $3 / 4$ of its original price?
a. $16 \%$
b. $37 \frac{1}{2} \%$
c. $45 \%$
d. $55 \%$
e. $621 / 2 \%$
20.Ann is 8 inches taller than Bob. If Ann casts a 25 -inch shadow when Bob casts a 30-inch shadow, how tall is Ann?
a. 40 in
b. 48 in
c. 80 in
d. 93.75 in not possible
e. This is
21.A bouncing ball loses $15 \%$ of its height on each successive bounce. If the ball is released from a height of 6 feet, how many bounces until it makes a bounce of 1 foot?
a. 7
b. 9
C. 11
d. 14
e. 40
20. A triathlete averages 2 mph during her swim, 18 mph during her bike, and 7 mph during her run. The running distance of a race is 14 times longer than the swimming distance and 50 miles shorter than the biking distance. If she finishes the race in 6 hours, 52 minutes, and 30 seconds, what is the total distance of the race?
a. 65 miles
b. 86.25 miles
c. 185.625 miles
d. 53 miles
e. 20.9 miles
21. In a poll of your neighborhood, you find that $40 \%$ of the families own a dog, $35 \%$ own a cat, and $50 \%$ own neither one. What percent of the dog owners do not own a cat?
a. $5 \%$
b. $15 \%$
c. $37 \frac{1}{2} \%$
d. $45 \%$
e. $621 / 2 \%$
22. A classmate completes a project relating the height of their friends to their shoe size. Which of the following would you expect to see as a chart summarizing their data?

a

b

C

d

e
23. If it takes 3 workers 4 days to build 5 wagons, how many days will it take 12 workers to build 40 wagons?
a. 8
b. 12
c. 24
d. 26
e. 13
24. If $(2 x+y)^{2}=(x+2 y)^{2}$, which of the following must be true?
a. $x=0$ and $y=0$
b. $x=y$
c. $|x|=|y|$
d. $2 x+y=0$
e. $x+2 y=0$
25. For what value of $b$ does $x^{2}+b x+20$ have exactly one real solution?
a. 4
b. $\sqrt{5}$
C. $4+\sqrt{5}$
d. $4 \sqrt{5}$
e. $\sqrt[4]{5}$
26. Which of the following has the same solution set as $(2 x-7)^{2}<25$ ?
a. $2 x-7<5$
b. $|2 x-7|<5$
C. $4 x^{2}+49<25$
d. $2 x-7<12.5$
e. None of these
27. Find the sum of the solutions for $\left|x^{2}-13 x\right|=30$
a. -26
b. -11
c. 0
d. 11
e. 26
28. The notation $f^{n}(x)$ means to compose $f(x)$ with itself $n$ times.

For example, $f^{4}(x)=f(f(f(f(x))))$. If $f(x)=|x-9|-|2 x-5|$, find $f^{50}(3)$.
a. -1
b. 3
c. 5
d. 250
e. $5^{50}$
31. Which of the following relations is not a function?
a. A relation that contains $(0,0)$ and $(1,1)$.
b. A relation that contains $(-2,3)$ and $(3,-2)$.
c. A relation that contains $(5,7)$ and $(6,7)$.
d. A relation that contains $(4,2)$ and $(4,3)$.
e. A relation that contains $(-3,-2)$ and $(3,2)$.
32. The following table gives the maximum number of regions that can be formed by cutting a circle with $n$ straight-line cuts. How many cuts are necessary to get at least 500 regions?

| Cuts | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| Regions | 2 | 4 | 7 | 11 |

a. 23
b. 32
c. 123
d. 231
e. 321
33. What is the maximum value of a quadratic function that has $x$-intercepts of $k,-2 k$ and a $y$-intercept of $3 k ?(k>0)$
a. $-\frac{k}{2}$
b. $\frac{27 k}{8}$
C. $\left(\frac{3 k}{2}\right)^{3}$
d. $3 k$
e. $-6 k^{3}$
34. Which is the graph of $|x|+|y|=4$ ?

35. The following graph shows the number of gallons of water in a tank which is both filled and emptied through the same pipe. At what time is the water flowing fastest through the pipe?


