Algebra II / Integrated Math III

2011

Sponsored by the Indiana Council of Teachers of Mathematics

Indiana State Mathematics Contest

This test was prepared by faculty at **Indiana State University**

ICTM Website
http://www.indianamath.org/

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Next year's math contest date: April 28, 2012

1)   Solve for x: $\dfrac{x-5}{x+2} \le 0$

A) $x \ge 5$ or $x \le -2$

B) $x \ge 5$ or $x < -2$

C) $-2 \le x \le 5$

D) $-2 < x \le 5$

E)   none of these

2)   The sum of the solutions to the equation $x - 2 = \sqrt{2x-1}$ is:

A)   4

B)   5

C)   6

D)   7

E)   none of these

3)   Write the following without a radical in the denominator: $\dfrac{-3}{-\sqrt{2}+\sqrt{5}}$

A) $-\sqrt{2}+\sqrt{5}$ B) $-\sqrt{2}-\sqrt{5}$ C) $\sqrt{2}+\sqrt{5}$ D) $\sqrt{2}-\sqrt{5}$ E) none of these

4)   What conic does the equation $4x^2 - y^2 - 8x - 4y - 9 = 0$ represent?

A) Circle   B) Ellipse   C) Hyperbola   D) Parabola   E) none of these

5)   The conic section with the smallest eccentricity is:

A) Circle   B) Ellipse   C) Hyperbola   D) Parabola   E) none of these

6)   The inverse of the function $f(x) = x^2, x \leq 0$ is:

     A) $y = -\sqrt{x}$ B) $y = \sqrt{x}$ C) $y = -x^{-2}$ D) $y = x^{-2}$ E) none of these

7)   The sum of the mean, median, and mode of the data: 80, 82, 82, 82, 84, 86, 88, and 88 is

     A) 246   B) 247   C) 248   D) 249   E) 250

8)   Solve the equation for r in terms of s and t: $s = 2t(r - 2\pi)$

     A) $\dfrac{s - 2\pi}{2t}$ B) $\dfrac{s}{2t} + 4\pi$ C) $\dfrac{s - 4\pi}{2t}$ D) $\dfrac{s}{2t} - 2\pi$ E) none of these

9)   The inverse of the function $y = \dfrac{1}{2}\ln(x - 3)$ is:

     A) $y = e^{2x+3}$

     B) $y = e^{2x-3}$

     C) $y = e^{2x} + 3$

     D) $y = e^{2x} - 3$

     E)   none of these

10)   Which property is illustrated by the statement: $2x + 7x = (2 + 7)x = 9x$?

     A)   Associative property of addition
     B)   Associative property of multiplication
     C)   Commutative property of addition
     D)   Commutative property of multiplication
     E)   none of these

11)   Consider the equation: $x^5 + 8x = 6x^3$. The sum of all the roots of this equation is:

     A) 0   B) $\sqrt{2}$ C) 2   D) $2\sqrt{2}$ E) 4

12)   Write an equation of the line that is tangent to the circle $x^2 + y^2 = 25$ at (-3, 4).

A) $3x + 4y = 25$

B) $3x - 4y = 25$

C) $-3x + 4y = 25$

D) $-3x - 4y = 25$

E)   none of these

13)   Simplify the following rational expression:

A) $x^4 + y^4$ B) $x^4 - y^4$ C) $x^4 + x^2y^2 + y^4$ D) $x^4 - x^2y^2 + y^4$ E) none of these

14) $3\log_5 25 + \log_{25} 625^{10} =$

A) 56   B) 46   C) 36   D) 26   E) none of these

15) $4^{1/4} \times 256^{1/4} =$

A) $4\sqrt{2}$ B) $5\sqrt{2}$ C) 8   D) 10   E) none of these

16)   Find the sum of the first ten terms of the following geometric series $\frac{1}{2} + \frac{1}{6} + \frac{1}{18} + \frac{1}{54} + \frac{1}{162} + \cdots$

A) $\frac{1}{2}\left(1 + \frac{1}{3^{10}}\right)$ B) $\frac{1}{2}\left(1 - \frac{1}{3^{10}}\right)$ C) $\frac{3}{4}\left(1 + \frac{1}{3^{10}}\right)$ D) $\frac{3}{4}\left(1 - \frac{1}{3^{10}}\right)$ E) none of these

17)   A function f is *odd* if $f(-x) = -f(x)$. Which of the trigonometric functions is NOT *odd*?

A) $y = \sin(x)$

B) $y = \csc(x)$

C) $y = \tan(x)$

D) $y = \cot(x)$

E) $y = \sec(x)$

18)   If the graphs of the lines with equations $bx - 2y + 7 = 0$ and $7x + ay - 2 = 0$ are perpendicular, then $\dfrac{a}{b}$ is equal to:

A) $-\dfrac{7}{2}$ B) $\dfrac{7}{2}$ C) $-\dfrac{2}{7}$ D) $\dfrac{2}{7}$ E) none of these

19)   The sum of the solutions of the equation $|2x + 5| = |3x + 25|$ is:

A) -20 B) -26 C) -24 D) -6 E) none of these

20)   Find the matrix A if the inverse of the matrix, $A^{-1} = \begin{pmatrix} 2 & -1 \\ 3 & 5 \end{pmatrix}$

A) $\dfrac{1}{7}\begin{pmatrix} 5 & -1 \\ 3 & 2 \end{pmatrix}$ B) $\dfrac{1}{7}\begin{pmatrix} 5 & -1 \\ -3 & 2 \end{pmatrix}$ C) $\dfrac{1}{13}\begin{pmatrix} 5 & -1 \\ 3 & 2 \end{pmatrix}$ D) $\dfrac{1}{13}\begin{pmatrix} 5 & -1 \\ -3 & 2 \end{pmatrix}$ E) none of these

21)   If the system $\begin{cases} 4x^2 - 9y^2 = 36 \\ y = x + k \end{cases}$ has exactly one solution, then k is equal to:

A) ± 1 B) $\pm\sqrt{2}$ C) $\pm\sqrt{5}$ D) ± 5 E) none of these

22)   If $f(x) = \dfrac{4x^3 + 3x^2 + 2x + 1}{2x - 2}$ and $i = \sqrt{-1}$ then the value of $f(i)$ is:

A) $i - 1$ B) $1 + i$ C) $-i$ D) i E) none of these

23)   The sum of the solutions to the equation $12 - 8 \cdot \log_2 x + (\log_2 x)^2 = 0$ is:

A) 68   B) 256 C) 4   D) 64   E) none of these

24) The maximum value of the function $y = -3 + 2\cos(5x - 2)$?

     A) -1   B) 2   C) 5   D) 7   E) none of these

25) The range of the function $y = -3\sqrt{1 - 2x} + 1$ is:

     A) $y \geq \frac{1}{2}$ B) $y \leq \frac{1}{2}$ C) $y \geq 1$ D) $y \leq 1$ E) none of these

26) Which equation is an *xy*-equation for the parametric equations $x = 5t - 9$ and $y = -3t + 13$?

     A) $y = \frac{5}{3}x - \frac{38}{3}$ B) $y = \frac{5}{3}x + \frac{92}{3}$ C) $y = -\frac{3}{5}x + \frac{92}{5}$ D) $y = -\frac{3}{5}x + \frac{38}{5}$ E) none of these

27) If $f(x) = 2x + 7$ and $f(g^{-1}(x)) = 9 - 2x$, then $g(x)$ is equal to:

     A) $-x - 1$ B) $-x + 1$ C) $x - 1$ D) $x + 1$ E) none of these

28) The graph of $4x^2 - 9y^2 + 16x - 144y - 560 = 0$ is

     A) a circle

     B) an ellipse

     C) a parabola

     D) a hyperbola

     E) two intersecting lines

29) How many points of intersection do the graphs of the following equations have?

     $4x^2 + 9y^2 - 16x + 54y + 61 = 0$ and $y^2 - x^2 + 6y + 4x + 4 = 0$

     A) 0   B) 1   C) 2   D) 3   E) 4

30) How many real and imaginary solutions does the polynomial $p(x) = x^5 - 3x^4 + 2x^3 - 6x^2$ have?

    A)   3 real solutions and no imaginary solutions

    B)   2 real solutions and two imaginary solutions

    C)   3 real solutions and two imaginary solutions

    D)   2 real solutions and no imaginary solutions

    E)   none of these

31) Let A and B be matrices with the same dimensions and let c be a scalar. Which of the following statements are true?

        I.   A + B = B + A

        II.   AB = BA

        III. c(A − B) = cA − cB

  A) I only   B) I and II only   C) I and III only   D) II only   E) I, II, and III

32) Mr. Kim waters one of his four plants every 6 days, another plant every 10 days, and the other plants every 14 and 21 days respectively. If he waters all four plants today, when is the next time the four plants will be watered on the same day?

    A)   210 days

    B)   420 days

    C)   360 days

    D)   480 days

    E)   none of these

33) When two six-sided dice are tossed, what is the probability that the sum of the two dice is greater than or equal to 10?

A) $\dfrac{1}{12}$ B) $\dfrac{1}{9}$ C) $\dfrac{5}{36}$ D) $\dfrac{1}{6}$ E) none of these

34)   A pizza store offers 13 toppings to choose from. How many different large pizzas can you order with exactly 3 different toppings? (Note: You can order a large pizza with 3 different toppings such as pepperoni, mushroom, and cheese, but not double cheese and mushroom)

A) 286   B) 572   C) 858   D) 1716   E) none of these

35)   There are three different boxes A, B, and C. Box A weighs 5 times as much as box C. Box C weighs $\dfrac{3}{4}$ as much as box B. Box B weighs 15 pounds more than box C. How much does box A weigh, in pounds?

A) 300   B) 225   C) 125   D) 75   E) none of these

36)   The diagram below represents the only possible paths for trips between cities A, B, C, and D. For instance, there are only five trip paths from city B to city C. How many different round-trip paths are there between A and D such that each round-trip passes **ONLY ONCE** through the location represented by the asterisk (that is, a round trip goes from A to B, to C, to D, to C, to B, to A)?
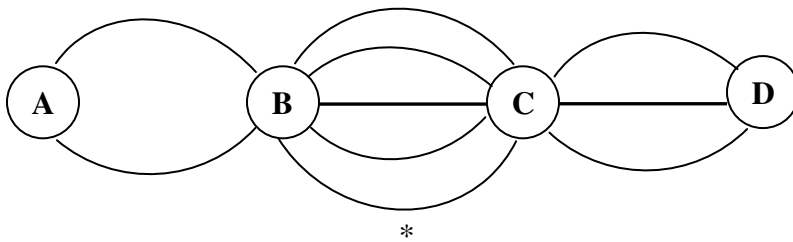
A) 144   B) 324   C) 576   D) 864   E) none of these